
Design For Variation

NASA Statistical Engineering Symposium

Williamsburg, VA 5/5/2011

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Pratt & Whitney, East Hartford, CT



Pratt & Whitney Engineering

A Passion for innovation

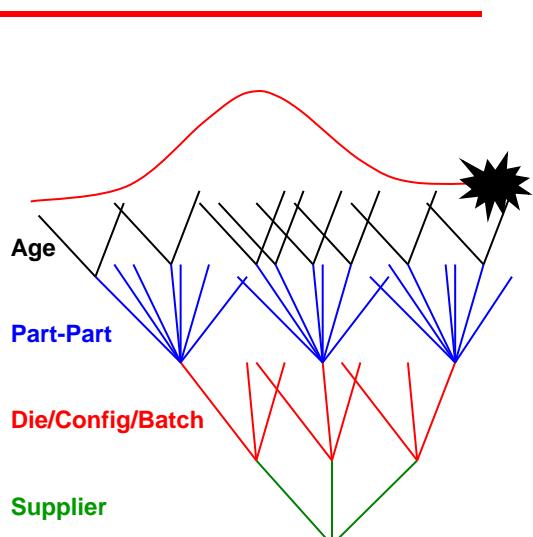


Design For Variation

A Strategic Initiative at Pratt & Whitney

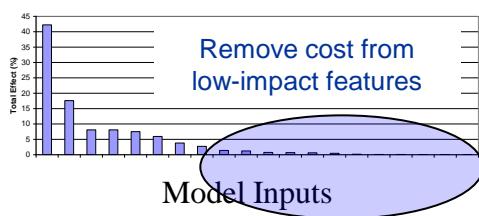
▲ To Reduce Escapes (Safety)

- Variation plays a significant role in field problems
- Cost of finding/correcting problems increases rapidly as product matures



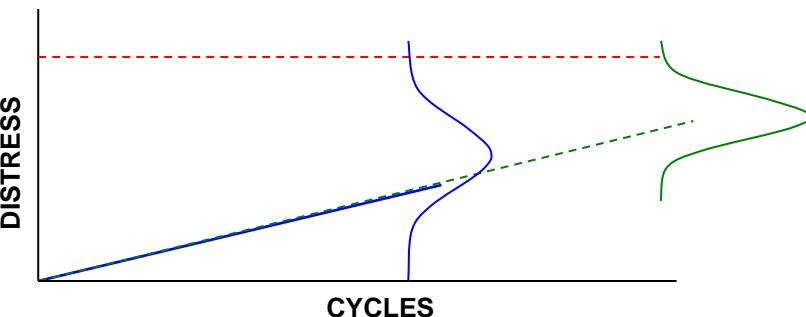
▲ To Improve Productivity (Cost/Competitiveness)

- Find and focus on important features (few?)
- Relax requirements on unimportant features (many?)
- Use Robust Design to reduce sensitivity



▲ To Maximize Rotor Life (Time on Wing)

- Rotor life depends on max distress / min life airfoil
- ‘Weakest link’ structure pervasive in gas turbines
- Reducing variation increases rotor life



Design For Variation (DFV) Strategic Plan

Vision: All Key Modeling Processes will be DFV-enabled

▲ Strategy

- Identify Key Processes
- Define elements of a DFV-enabled modeling process
- Provide Resources under Strategic Initiative

Mechanical Systems and Externals

Carbon Seal Performance
Ball & Roller Bearing Design
FDGS Durability

Externals: Forced Response Analysis

Combustor and Augmentor

Combustor pattern factor
Combustor Liner TMF
Augmentor Ignition Margin Audit
Mid Turbine Frame Robust Design

Air Systems

Thermal Management Model
Internal Air System Model
Engine Data Matching

Black: Legacy Task
Green: 2010 funded
Blue: 2011 funded (new)

DFV Infrastructure (Statistics & Partners)

Sens / Uncert / Opt Software

High Perf Computing

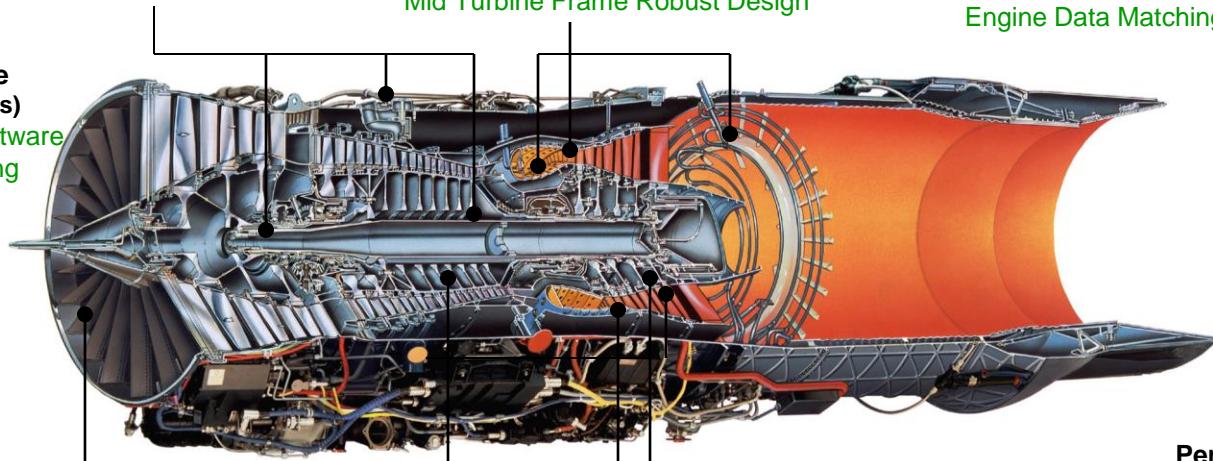
Training

ESW

Communications

Input Data

Tech Support



Fan & Compressor

HFB Productivity
Parametric Airfoil

Compressor Aero Design

Structures

Probabilistic HCF

Parametric Geometry Simulation Model
Engine Dynamics and Loads

Turbine

Turbine Blade Durability
Turbine Vanes and BOAS Durability
Rotor Thermal Model
Airfoil LCF Lifing

Validation Testing
Engine Validation Planning

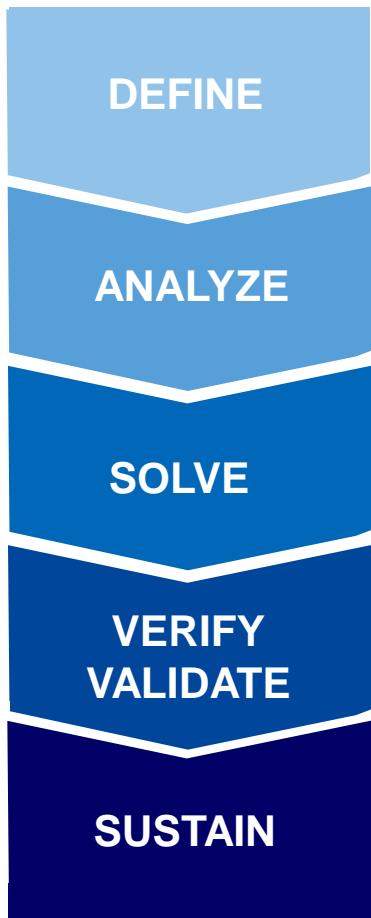
Vehicle Systems
Probabilistic
Ambient Temp
Distribution

Performance Analysis

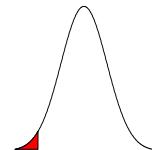
Performance Monte Carlo Risk Assessment
Engine Test Confidence, Uncertainty
Uncertainty in Engine System Predictions
Production Test Data Trending and Analysis

Design For Variation

Five Components



DEFINE Customer requirements (probabilistic)



ANALYZE Identify root causes of variation and uncertainty, develop variability/uncertainty model

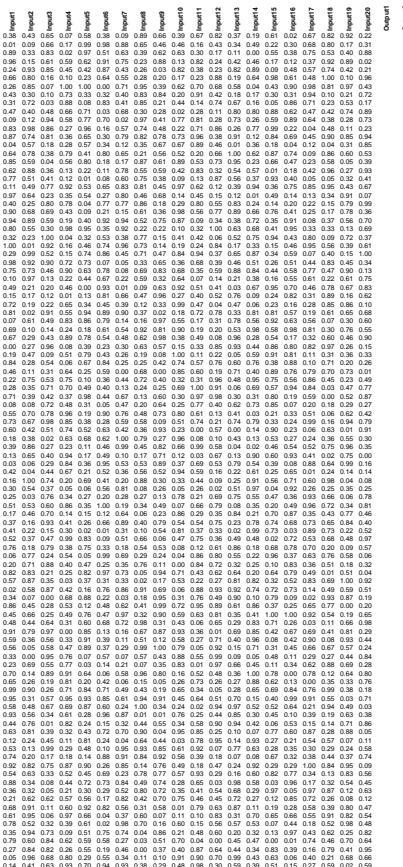
SOLVE Identify ‘optimum’ design that satisfies requirements

VERIFY/VALIDATE Variability/Uncertainty model

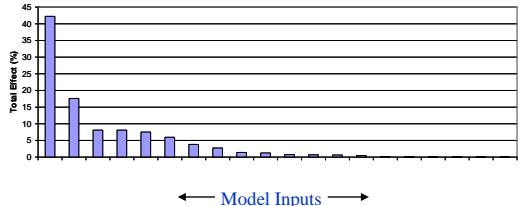
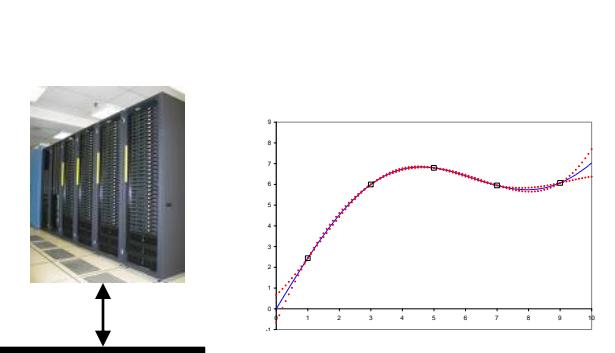
SUSTAIN Stable system of causes of performance variation

Design For Variation

ANALYZE Identify root causes of performance variation and uncertainty and their effects



Engineering Model



Develop Model Emulator, Sensitivity Analysis

Run Experiment Through Engineering Model

Design Space Filling Experiment Over Model Input Space

Refine Distributions of Important Model Inputs

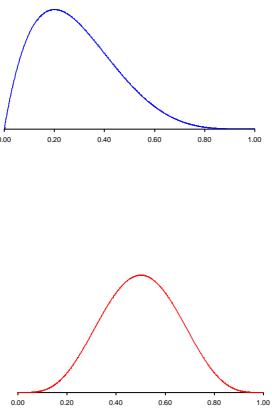
Perform Bayesian Model Calibration

Run Real World Uncertainty Analysis

- Parameter uncertainty update
 - Bias correction
 - Residual variation

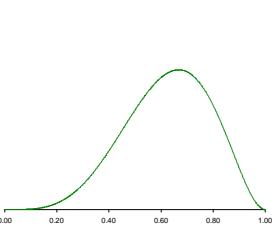
Accounting for uncertainty in

- Model input
- Model itself



Real-World Validation Data

Bayesian Model Calibration

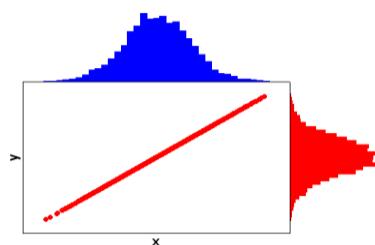


Design For Variation

SOLVE Identify optimum design that satisfies requirements

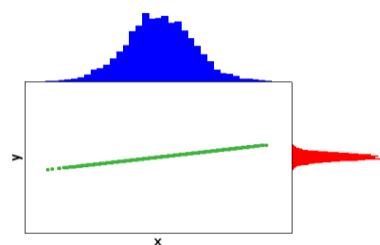
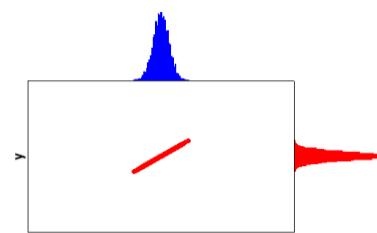
- ▲ Performance characteristic $y = f(x_1, x_2, \dots, x_p)$ depends on p inputs
- ▲ The variance of y can be approximated by

$$\sigma_y^2 \approx \left(\frac{\partial f}{\partial x_1} \right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f}{\partial x_2} \right)^2 \sigma_{x_2}^2 + \dots + \left(\frac{\partial f}{\partial x_p} \right)^2 \sigma_{x_p}^2$$



- ▲ We can reduce σ_y^2 by

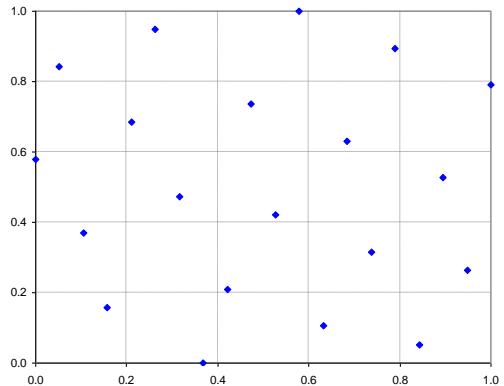
1. Reducing $\sigma_{x_i}^2$: the variance in the inputs x_1, x_2, \dots, x_p
2. Reducing $\frac{\partial f}{\partial x_i}$: the sensitivity of y to variation in x_1, x_2, \dots, x_p



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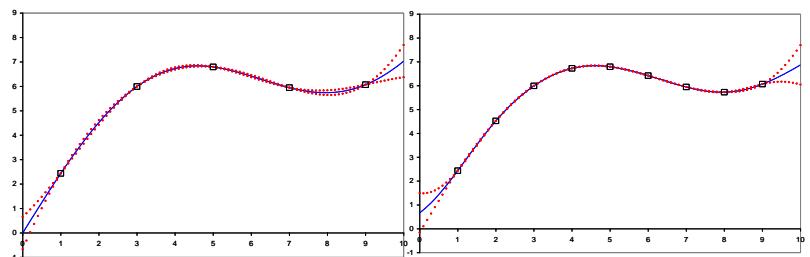
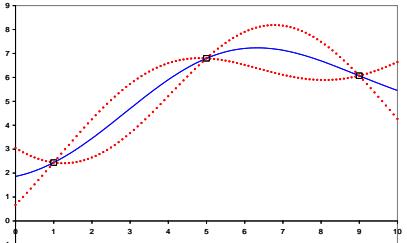
ANALYZE : Key Technologies

1. Latin Hypercube Experimental Designs

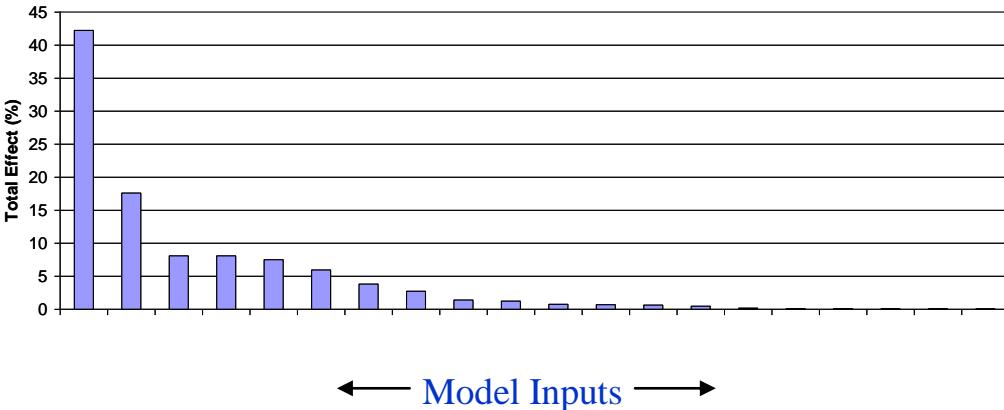


2. Gaussian Process Emulators

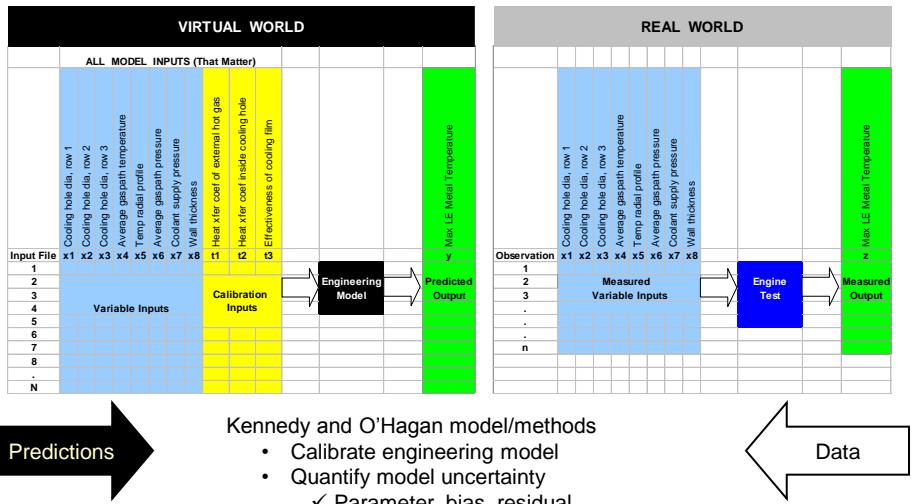
Simple function
 $f(x) = x + 3\sin(x/2)$



3. Variance-Based Sensitivity Analysis

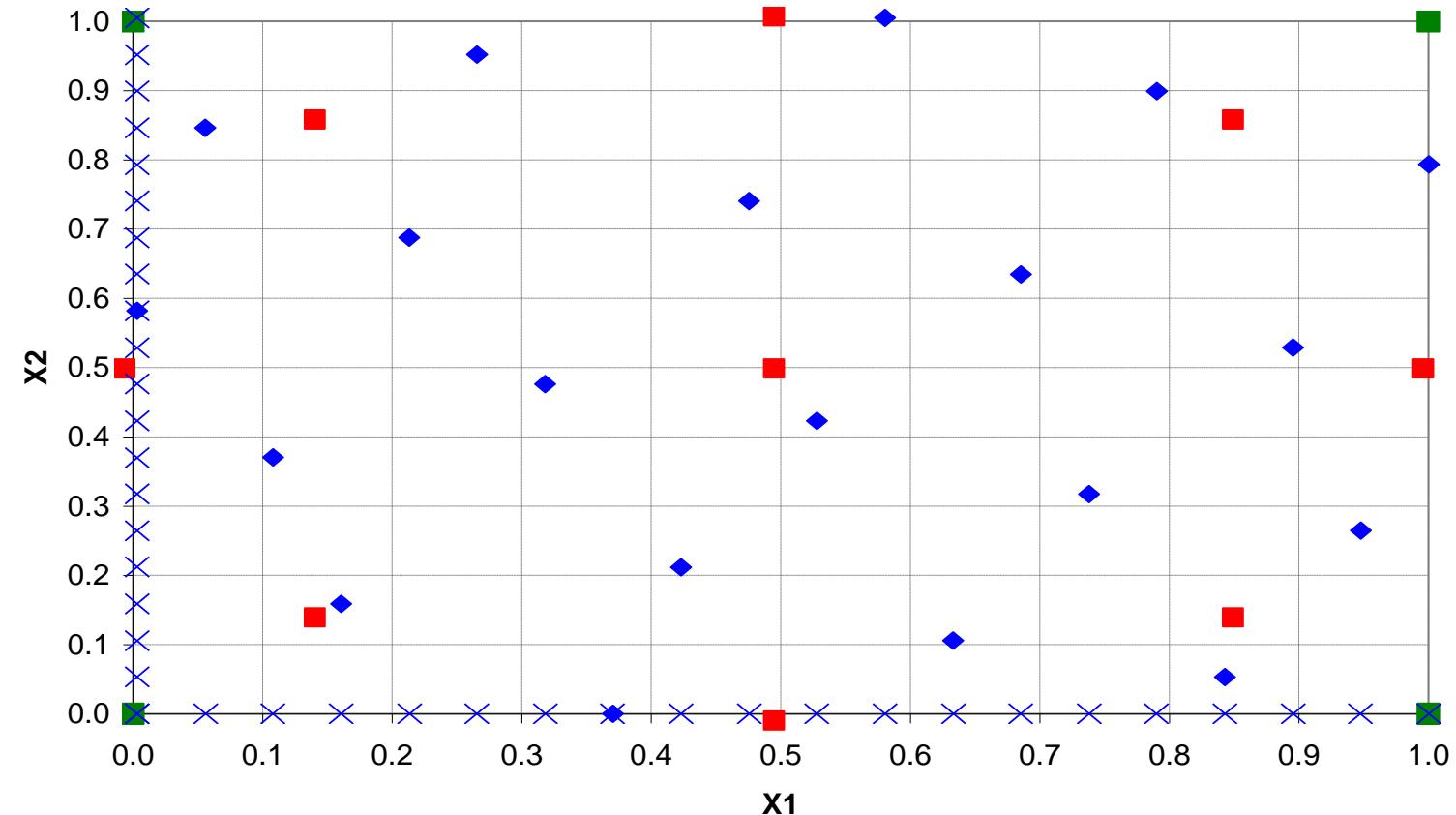


4. Kennedy and O'Hagan Bayesian Model Calibration



Design For Variation

1. Latin Hypercube Experimental Designs



2-level factorial designs assume linearity and focus on vertices of design space

2nd order response surface designs like the CCD tend toward corners and edges but improve on factorial designs

Latin Hypercube samples are space-filling and guarantee uniform distribution over margins (see X's in diagram)

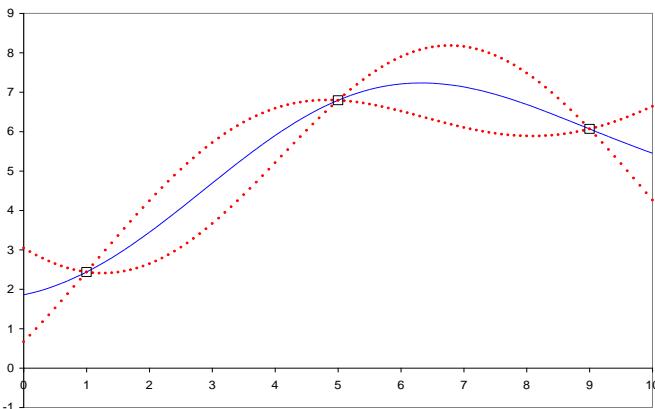
Design For Variation

2. Gaussian Process Emulators

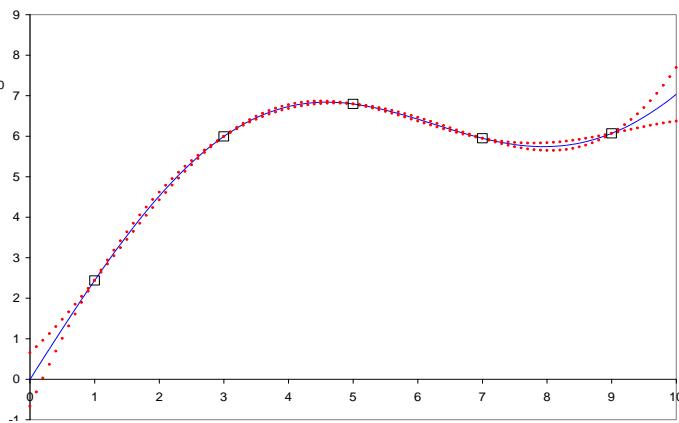
- ▲ Thousands of model runs typically required for uncertainty analysis
 - Calibration
 - Propagation of scenario uncertainty
 - Sensitivity analysis
- ▲ Not practical for computationally expensive codes
- ▲ Gaussian process models as ‘emulators’
 - Approximate model $y=f(x)$
 - Provide probability distribution quantifying uncertainty at new design points

Design For Variation

2. Gaussian Process Emulators

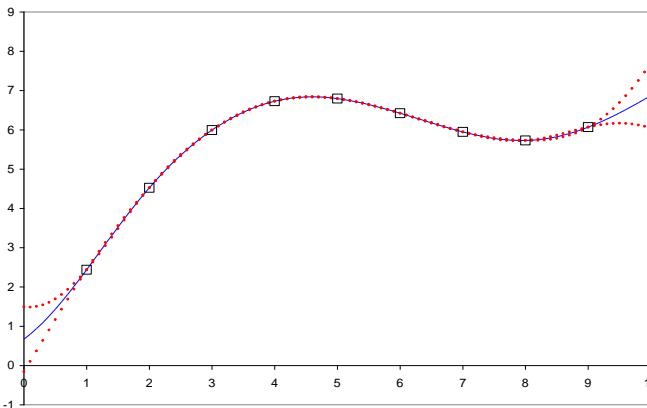


Simple function
 $f(x) = x + 3\sin(x/2)$



- Observations**
1. Zero uncertainty at training data
 2. Uncertainty increases with distance from training data
 3. Uncertainty decreases as training data are added
 4. Emulator shape changes as it “learns” from training data

These are desirable properties of any emulator



Ref: O'Hagan, A. (2006). Bayesian analysis of computer code outputs: a tutorial. Reliability Engineering and System Safety, 91, 1290-1300.

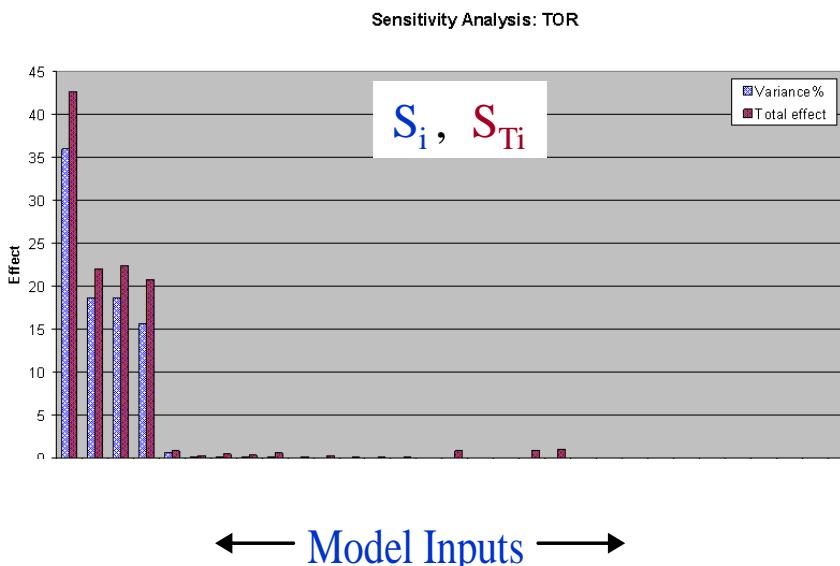
Design For Variation

3. Variance Based Sensitivity Analysis

How much of the variance of model output is due to each input?

1. $S_i = \text{Var}[E(Y|X_i)]/\text{Var}(Y)$
 - % Due to Main Effect of X_i

2. $S_{Ti} = E[\text{Var}(Y|X_{-i})]/\text{Var}(Y)$
 - % Due to Total Effect of X_i
 - Main Effects + All Interaction Effects involving X_i , if X_i independent



Design For Variation

4. Kennedy and O'Hagan Bayesian Model Calibration

What is an Uncertainty Analysis?

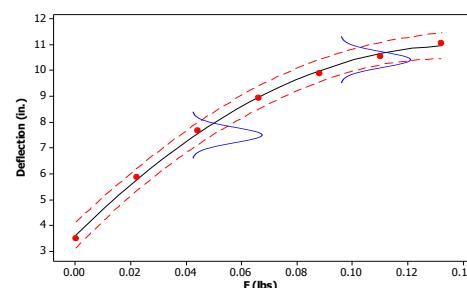
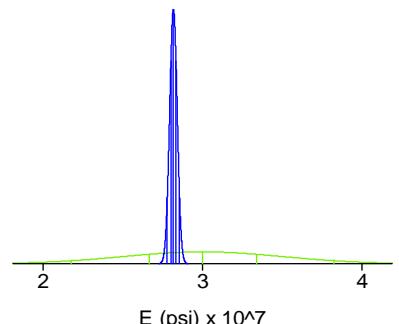
- ▲ An uncertainty analysis augments a single point prediction with a probability distribution that accounts for
 - Variability or uncertainty in model input
 - Capability of model
- ▲ How variable or uncertain is your model input
 - Uncertainty due to random variation in model inputs
- ▲ How capable is your model
 - Uncertainty due to lack of agreement between model predictions and physical measurements (the real world response)
 - Model validation

4. Kennedy and O'Hagan Bayesian Model Calibration

Uncertainty

- ▲ Some sources of uncertainty in the state of a physical system associated with a deterministic prediction

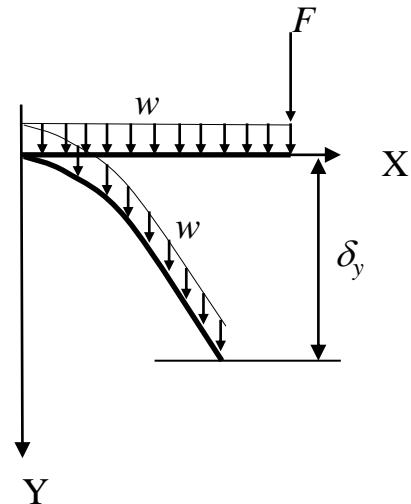
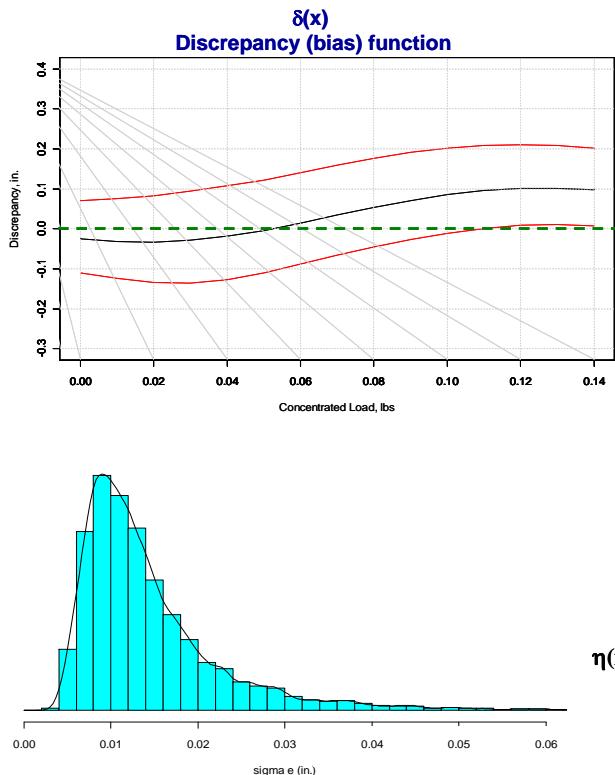
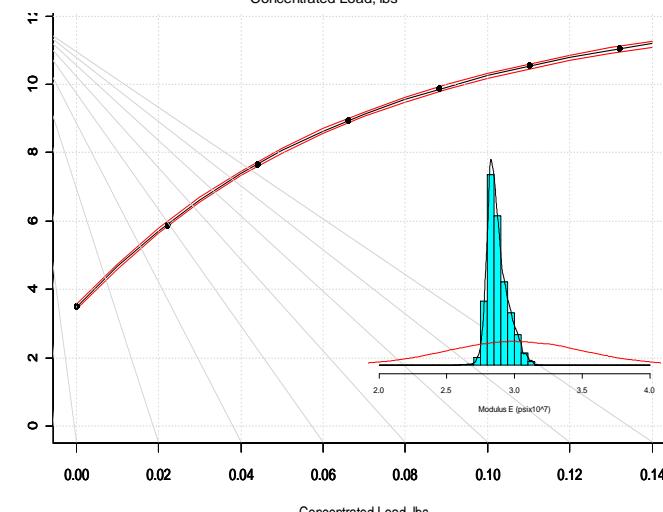
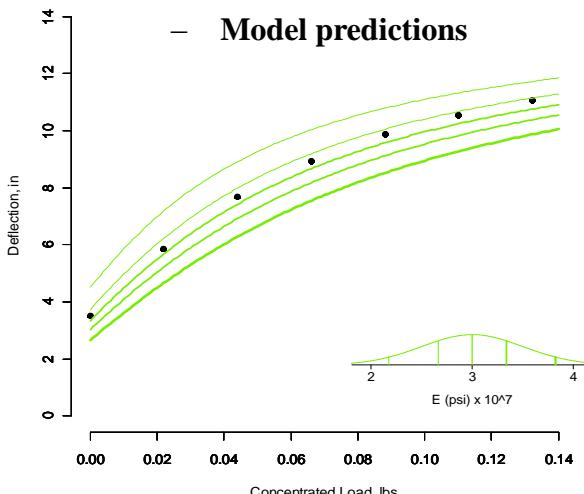
- Scenario uncertainty - Uncertainty about some future measurable values of model inputs, e.g., what missions *will* be flown, what hole sizes *will* result from the laser drilling process
- Parameter uncertainty - Uncertainty about best values of model parameters (e.g. heat transfer coefficients, Young's modulus, compressor efficiency) or uncertain inputs (e.g. boundary conditions)
- Model structure uncertainty - Uncertainty about the difference between the mean of the real world process being modelled and the model prediction using the best possible parameter values. Sometimes referred to as model inadequacy, model discrepancy, or model bias.
- Residual variation - Variation in real world outcomes at a given (known) scenario, due to variation in factors that are outside the model or measurement error



4. Kennedy and O'Hagan Bayesian Model Calibration

A statistical framework for combining experimental data with model predictions to provide best estimates and uncertainty for

- Model calibration parameters
- Systematic discrepancies between model and data
- Standard deviation of random discrepancies between model and data
- Model predictions



Nomenclature

x	Model variable (measurable) inputs (exper. conditions)
θ^*	Model calibration inputs
θ	Best value of model calibration inputs
$\zeta(x)$	True average system response given inputs x
$\eta(x, \theta^*)$	Model prediction for inputs x and θ^*
$y(x)$	Experimental observation for inputs x
$\delta(x)$	discrepancy (bias) between $\zeta(x)$ and $\eta(x, \theta)$
$e(x)$	random observation error of the experimental data
$y(x) = \zeta(x) + e(x)$	
$\zeta(x) = \eta(x, \theta) + \delta(x)$	
$y(x) = \eta(x, \theta) + \delta(x) + e(x)$	

Data few and noisy but unbiased

Model smooth but biased

Combine to get best from both

Design For Variation

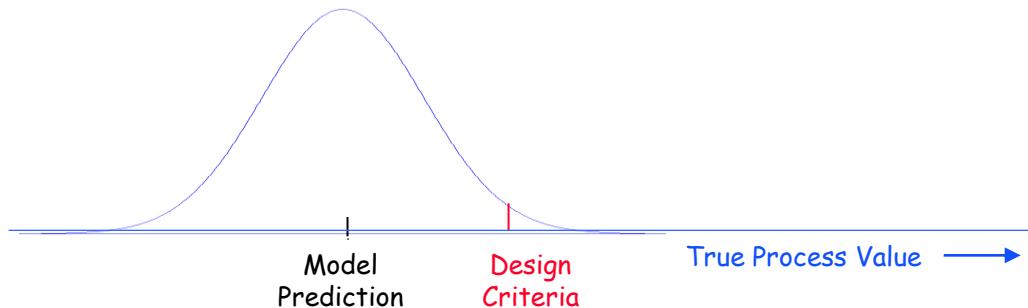
Systematic Process for Designing for and Managing Uncertainty and Variability

- ▲ Establish probabilistic design requirements
- ▲ Emulate, calibrate engineering models
- ▲ Solve for design that meets probabilistic requirements
 - Look for opportunities for making design less sensitive to variation
- ▲ Validate and sustain model
- ▲ Write Engineering Standard Work, develop local training

Design For Variation

- ▲ Goal: quantify, understand, and control the risk of not meeting design criteria or exceeding thresholds

- ▲ “The revolutionary idea that defines the boundary between modern times and the past is the mastery of risk: the notion that the future is more than a whim of the gods and that men and women are not passive before nature.”
 - Peter Bernstein, “*Against the Gods: The remarkable story of risk*”



Bayesian Model Calibration

Challenges

- ▲ Establishment of ‘Gold Standard’ numerical methods
- ▲ Commercial software availability
- ▲ Parametric geometry
- ▲ Optimal [model:experimental] DOE for model validation
- ▲ Computational issues (e.g. matrix inversion $O(n^3)$)
- ▲ Large transient models
- ▲ Calibration data outside operational range
- ▲ What if only sub-models can be calibrated?
- ▲ Discrepancy root cause investigation structure
 - Original research assumed measurement process free of bias
 - Sometimes instrumentation technology can rival model technology
- ▲ Best approach to confounding issues
- ▲ Lack of textbooks, engineering methods and applications papers

Design For Variation

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Applications

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Kennedy and O'Hagan Bayesian Model Calibration

Sample of Available Software [Preliminary]

Software	Data Analysis	Bayesian Data Analysis	Space-Filling DOE	Model Emulation*	Sensitivity Analysis	Calibration, single response	Calibration, multiple responses	Optimization	Uncertainty Analysis
GEMSA			X	X	X				X
GEMCAL						X			
GPMSA			X	X	X	X	X		
DAKOTA			X	X	X			X	X
Isight			X					X	X
Matlab (Optimization Toolbox)									X
Matlab (Statistics Toolbox)	X		X						X
R (BACCO Package)				X	X	X			
R (Base Package)	X							X	X
R (gptk Package)				X					
R (lhs, DiceDesign, DiceKriging Packages)			X						
R (mlegp Package)				X	X				
R (tgp Package)				X	X				
R (Sensitivity Package)					X				
R (LearnBayes package)		X							
WinBUGS		X							
JMP	X		X	X	X				X
SAS	X	X						X	X
Minitab	X								
SimLab (Matlab/R)					X				X

MUCM Toolkit (algorithms): <http://mucm.aston.ac.uk/MUCM/MUCMToolkit/index.php?page=MetaHomePage.html>

*Various Independent softwares exist for Model Emulation. See <http://www.gaussianprocess.org/> for a listing

Note: GPMSA available through Brian Williams at LANL: brianw@lanl.gov